

On Crises in Financial Markets

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Abstract: The reasons of the financial markets collapse and methods of their forecasting are investigated in this article. A model based on hypothesis of the quantum nature of the impact of information on financial markets is given. It is shown that in information-saturated volatile financial markets, sharp price jumps are really expected

Motivation for this research is inability of traditional approach for explaining sharp price jumps during financial crises. They are unexpected according to the traditional theories. When considering the logarithm of relative price changes over

the period $y_{t_k} = \ln\left(\frac{N_{t_k}}{N_{t_{k-1}}}\right)$ it was found that the statistical characteristics of this random value differ from the characteristics

of the normal distribution. The approach, developed in this paper, explaining the possibility of sharp price jumps, seems to be more harmonious than the traditional approach.

Novelty of given approach consists in considering a model based on the assumption about the quantum (discontinuous) nature of information impact on financial markets. The process of information transfer is quantum – i. e. the information is transmitted in portions, multiples of a quantum of information. There are discrete information levels. When moving from one level to another, it is necessary to absorb or emit one quantum of information. Thus, the amount of information of a particular level is necessarily a multiple of the quantum of information.

Methodology and methods are based on probability and differential equations. Equation with respect to logarithm of increment of prices $y = \ln\left(\frac{N(t_0+\Delta t)}{N(t_0)}\right)$ is thoroughly investigated. The probability density function for each information price level $P_n(y) = \Psi_n^2(y)$, where $\Psi(y)$ is called the wave function of prices. Equation with respect to $\Psi(y)$ is thoroughly investigated too.

There are many calculations of various probabilities and other characteristics of y (logarithm of prices increment) for different information price level. The hierarchy of information-price levels is autonomous – i.e. each of them has its own separate probability characteristics, different functions of probability density distribution. The normal distribution takes place only when $n=0$. For all others $n=1, 2, 3...$ the density functions are different from Gaussian.

Keywords: Financial markets, asset prices, price emissions, risks, the quantum nature of information, resonance phenomena, density wave function, quantum oscillator.

I. INTRODUCTION

A lot of works performed by both financial analysts and mathematicians are devoted to the study of the causes of financial market failures and methods of their forecasting. The theme itself sometimes provokes spectacular statements and conclusions, often without any convincing grounds. However, there are a number of serious approaches that have yielded encouraging results in recent years. Most of them, one way or another connected with econophysics – the field of Economics, adjacent to physics. In line with this direction lies our research, developing an unexpected aspect of solving the problem

II. TRADITIONAL APPROUCH

In the 70s and earlier scientists mainly worked with data recorded at long intervals (year, quarter, month, week). Typical probabilistic-statistical models (for increments of logarithms of financial indices) were models of random prices, moving average, autoregression and their combination. All models were linear.

Nonlinear models appeared in the 80s, due to the emergence of technical capabilities of daily trading data analysis. The most well-known of these models are ARCH (autoregressive model of conditional inhomogeneity), GARCH (generalized autoregressive model) and their numerous modifications. In the 90-ies information and computer progress made it possible to analyze trade data coming almost continuously within the day. The possibility of almost continuous obtaining of information allowed us to study the statistics of the high-frequency nature of changes over time, to identify a number of specific features in the dynamics of financial indices: the nonlinear nature of their formation and the aftereffect, which is expressed in the fact that many price indices.... "remember" the past. When considering the logarithm of relative price changes over

the period $y_{t_k} = \ln\left(\frac{N_{t_k}}{N_{t_{k-1}}}\right)$ it was found that the statistical

characteristics of this random value differ from the characteristics of the normal distribution. The most significant arguments for deviation from "normality" are the difference between the skewness coefficient (skewness) and the excess coefficient (elongation coefficient) from zero. The presence of a positive skew coefficient means that the empirical density distribution is asymmetric with a steeper drop to the left than to the right. The existence of a too-large excess, growing with decreasing Δ (here $\Delta = t_k - t_{k-1}$; $k=1, 2... t$ - time;

$N_{t_k}; N_{t_{k-1}}$ - asset prices at moments t_k and t_{k-1}),

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determined by the fourth moment

$$\hat{m}_\alpha(Y) = \frac{1}{n} \sum_{i=1}^n (y_{t_i} - \bar{Y})^\alpha; \quad Eks(Y) = \frac{\hat{m}_4}{(\hat{m}_2)^2} - 3, \text{ indicates that}$$

the distribution of values $Y \{... y_{t_k}^\Delta ... \}$ has "heavy tails".

This should be understood as a slower decrease in the probability density function on the asymptotic compared to the normal distribution. The deviation from the normality of values y_{t_k} observed for exchange rates, stocks and other financial assets is confirmed not only by the type of empirical densities (histograms), but also by standard methods of statistics: χ^2 - test; criteria of agreement (for example, Pearson's criterion), the quantile method. Note that all sorts of discussions in the financial literature on the topic of "heavy tails" can be found in the works (Samuelson, 1965; Mandelbrot, 1969; Fama, 1965). These papers note that elongation and heavy tails arise, for example, in ARCH, GARCH models when considering mixtures of normal distributions. For example, hyperbolic distributions can be obtained by mixing normal distributions with different variances.

After the works of B. Mandelbrot (1967:72) and E. Fama (1965), models of financial indices based on stable distributions gained popularity in the financial literature. Their density function has their "stability index" α , which takes values from the half-interval (0,2].

In the case of $\alpha=2$ the distribution is normal; in the case of $0 < \alpha < 2$ the corresponding distribution is a Pareto distribution, where the "tail index" α is exactly the "stability index". One hundred years ago, the Italian economist Pareto investigated the statistical nature of individual incomes in a sustainable economy by modeling them using a distribution $y \sim x^{-\nu}$ where y is the number of people with income x or greater than x ; $\nu=1.5$.

The hypothesis of a stable distribution when $0 < \alpha < 2$ to describe $Y \{... y_{t_k}^\Delta ... \}$ is natural, since it is characterized by both heavy tails and elongation observed in statistical data.

The appeal to stable distributions is also justified by the characteristic property of self-similarity of these distributions: if X and Y are Independent and have a stable distribution with index α , then their sum also has a stable distribution with the same index or, that is the same, the composition (folding) of distributions X and Y is a distribution of the same type. In statistical analysis of financial time series, it has been observed that many of them have properties of statistical self-similarity, manifested in the fact that their "parts are arranged in the same way as the whole." For example, if S_n ($n=0,1,2,...$) are the daily values of the s&P500 index, then the empirical densities $\hat{f}_n(x)$ and $\hat{f}_{kn}(x)$, where ($k=2,3,4,...$) are found over a large number of quantities

$$Y_n \{... \ln(\frac{S_n}{S_{n-1}}) ... \} \text{ and } Y_{kn} \{... \ln(\frac{S_{kn}}{S_{k(n-1)})} ... \}$$

are such that $\hat{f}_n(x) \approx k^H \cdot \hat{f}_{kn}(k^H \cdot x)$, where H is a constant which (as opposed to the expected value according to the Central limit theorem) is significantly larger. For strictly α -stable

processes $H = \frac{1}{\alpha}$. Such properties require an explanation and it was given within the framework of the General concept of statistical self-similarity (fractality), which led not only to such important concepts as fractal Brownian motion, fractal Gaussian noise, but also had a decisive influence on the creation of fractal geometry (Mandelbrot 1969). The concept of self-similarity is closely connected with such improbability concepts and theories as chaos, nonlinear dynamical systems.

From an economic point of view, this is a natural requirement to preserve the nature of data distributions during time aggregation, the implementation of which for stable distributions makes their use justified.

However, when operating with stable distributions, a number of significant difficulties arise due to the following reason. If X is a random value with a stable index $0 < \alpha < 2$ distribution, then $M(x) < \infty$ only if $\alpha > 1$. In General, $M\{(x)^p\} < \infty$ if and only if $p < \alpha$. Here M is the expectation operator; $M\{(x)^p\}$ - the initial moment of the order p of the random variable x .

Thus, for a stable distribution with an index $\alpha \in (0,2)$, the tails are so "heavy" that the second moment (variance) $M\{x^2\}$ is infinite (the corresponding integral is divergent). This fact gives significant statistical difficulties (for example, in the analysis of the quality of various estimates, criteria based on the use of variance). And, on the other hand, it is difficult to explain economically and to actually verify this fact because, as a rule, only a limited number of statistics are available. It is clear that in connection with this circumstance, the assessment of the true value of the "tail index" α is quite delicate, not to say reliable enough.

This is due to the fact that for a "good" estimate of α it is necessary, on the one hand, to have a lot of observations in order to gain a significant number of extreme values, by which only the "tail effects" and "tail index" can be estimated. But on the other hand, the presence of a large number of "non-extreme" observations will introduce a bias in the estimation of the true value of α .

From the properties of stable distributions it follows that if we use them as densities of distributions of the stock indices, it is not possible to combine three requirements: preservation of type of distributions in composition, the presence of heavy tails with index $\alpha \in (0,2)$ and finiteness of the second moment and hence variance.

It is clear that finiteness of variance holds for Pareto-type distributions with a "tail index" $\alpha > 3$. Although such distributions do not have the property of closure with respect to the composition, they have an important property of preserving the character of the decrease of the density in the composition: if random values X and Y have the same Pareto distribution with index 2 and are independent, then their sum $X+Y$ also has a Pareto distribution with the same "tail index" 2. From this point of view, Pareto-type distributions can be considered as satisfying the desired property of "stability of the tail index" in composition.

From the above it becomes clear why to the index 2, which determines the behavior of distributions of quantities at infinity $\{... y_{t_k}^\Delta \dots\}$ (asymptotic), is given so much attention. This index can be given an economic and financial explanation. The tail index indicates, in particular, how active players with speculative interests are in the market. If the "tail index" α is large ($\alpha > 3$), it means that anomalous emissions in price values $Y_\Delta \{... y_{t_k}^\Delta \dots\}$ beyond the interval $(M(Y) - 2\sigma; M(Y) + 2\sigma)$ are rare. Here $\sigma(Y) = \sqrt{D(Y)}$ is the standard deviation value, where D is the variance operator). I. e. the market behaves "smoothly", without big fluctuations in values of the prices of distribution of increments of logarithms of the prices asymptotically tends to zero rather quickly that there were all moments at least to the 4th order. In this sense, the market at high values α can be considered as effectively (in the sense of the term, which gave it P. Samuelson) functioning. Thus, the value of the index α is a measure of the efficiency of the market.

Unfortunately, it should be noted that so far in the financial literature there is no consensus on what is still the true value of the "tail index" α for certain exchange rates, stocks and other financial instruments.

This is explained, as already noted, by the difficulty of constructing effective estimates $\hat{\alpha}_n$ (n – number of observations) of the parameter α . Thus, in (Guillaume, Dacorona, Dave, Muller, Olsen and Pictet, 1997) a table of values of the "tail index" $\alpha = \alpha(\Delta)$ of exchange rates of major currencies against USD is given, assuming that the Pareto distribution acts on the asymptotic. An important conclusion that follows from the analysis of the values of the table (compiled from a large database, and therefore seems reliable) is that (exchange rates) / USD have (for $\Delta = 10$ min) a Pareto distribution with an index $\alpha \approx 3,5$. At the same time, as the interval increases, the index grows to a value $\alpha \approx 4,0$. It becomes plausible that the variance of Y_Δ -finite (a property incredibly desirable!), although this cannot be said about the fourth moment, which determines the amount of elongation in the vicinity of the central values.

The statistical analysis of data on the s&P500 index is given in work (Guillaume *et al.* 1997). The evolution of the index on the NYSE (New York Stock Exchange)

for 6 years (from January 1984 to December 1989) was considered. A total of 1.447.514 ticks were registered. On average, the ticks were with a minute interval in 1984-85 and with a fifteen-second interval in 1986-87. The value of the estimate α obtained in (Mantegna and Stanley 1995) is as follows: $\alpha = 1.40$ 0.05. Although the estimates in (Mandelbrot 1967) and (Mantegna *et al.* 1995) were obtained under different hypotheses about the nature of the distributions: in (Guillaume *et al.* 1997) the hypothesis was that the distribution belonged to the "stable" type, in (Mantegna *et al.* 1995) – to the Pareto distribution, the discrepancy in the estimates is too large. After all, we cannot seriously assume that the internal state of the US economy, which is an indicator of the s&P500 index and the state of the world economy, characterized largely by quotations of currencies on FOREX against the us dollar are so different that their "tail indices" differ by several times.

III. QUANTUM NATURE OF INFORMATION TRANSFER

Summing up, we note that traditional approaches do not explain the sharp price emissions that reach far beyond the boundaries of the interval 2σ . If, for example, the random variable ξ is distributed according to Gauss's law ($\xi \in N(\alpha, \sigma^2)$), then knowing its value "today", its value "tomorrow" in 95.44% of cases will lie in the interval $(\xi_{n-1}(1-2\sigma); \xi_{n+1}(1+2\sigma))$ and it means that only about 2.3% of cases ξ_n will be more then ξ_{n-1} and 2.3% of cases - less then ξ_{n-1} . This serves as a trigger for panic in the financial markets, leading to huge monetary losses. At present, we cannot rely with full confidence on traditional approaches in predicting the pre-crisis conditions of the stock market.

As a model describing the price fluctuations of stock market assets, we propose the equation:

$$\ddot{y} + 2\delta\dot{y} + \omega^2 y = F_0 \cos \gamma t \quad (1)$$

where $\ddot{y} = \frac{d^2 y}{dt^2}$; $\dot{y} = \frac{dy}{dt}$ - respectively, the second and

first time derivatives: $\delta = \frac{q}{2}$, where q -the coefficient of

market friction; $\omega = \sqrt{\beta}$ - circular (cyclic) natural frequency of harmonic oscillation; parameter β - the coefficient of rigidity of the "market spring", on which depends the value of the "returning force" arising from the deviation of asset prices from the equilibrium position and directed to the equilibrium position;

$y(t) = \ln\left(\frac{N_n}{N_0}\right)$; $N_n = N(t_0 + \Delta t)$ - the cost of a single

asset at the time $n = t_0 + \Delta t$ provided that at the time t_0 to its price was N_0 ; F_0 – amplitude of external periodic "force" (information) (the degree of influence of information on the financial market); γ - frequency of fluctuations of external periodic information.

It is empirically established that under the influence of an external periodic force on the physical system, forced oscillations occur with the frequency of changes in the external force. Unlike Newtonian mechanics, by force we mean information that affects the price of market assets.

We distinguish 4 categories of available information.

1. Information contained in past price values.
2. Information caused by imperfect understanding of market participants (the actual course of events already includes the thinking of participants).
3. Information contained in publicly available sources (Newspapers, magazines, television, Internet, etc.);
4. Every piece of information imaginable.

To clarify the concept of "information" we will proceed from the fact that the uncertainty arising in the market can be characterized as randomness within a probability space (Ω, F, P) . Here Ω is the outcome space; F is the algebra of all possible subsets in Ω ; P is the probability measure on (Ω, F) .

The General solution of equation (1) has the form:

$$Y = Ae^{-\delta t} \cos(\omega_1 t + \phi) + B \cos(\gamma t + \nu) \tag{2}$$

Here $\omega_1 = \sqrt{\omega^2 - \delta^2}$; A, ϕ are constants of integration. The first term decreases exponentially in time, so that after a certain interval only the second term remains $Y = B \cos(\gamma t + \nu)$. At the same time

$$B = F_0 \cdot ((\omega^2 - \gamma^2)^2 + 4\delta^2 \gamma^2)^{-\frac{1}{2}}; \nu = \arctg(2\delta\gamma \cdot (\gamma^2 - \omega^2)^{-1})$$

Of the three cases, $\gamma \ll \omega; \gamma \gg \omega; \gamma \approx \omega$, the third possibility is of the greatest interest. It's a state of resonance. The amplitude of forced oscillations increases sharply when the frequency of the external "force" approaches the frequency of natural oscillations. At resonance, the asset price makes its own fluctuations (almost without friction), and external information only pushes them. The law of price fluctuations has the form:

$$Y = B \cos(\omega t) \tag{3}$$

$$\text{Here } \omega = \gamma; B = \frac{F_0}{2\delta\omega}$$

There is an important aspect that needs to be discussed in connection with the model of financial market fluctuations presented here. The price increments $y_\Delta(t) = \ln(N(t + \Delta t)) - \ln(N(t))$ are realizations of a random function $Y(t) = X \cdot \cos(\omega t)$, where X is a random variable with a distribution different from the normal one.

The law of price fluctuations of the form (3) is the law characteristic of the excited market. It is well known, however, that in a calm, efficient financial market, the logarithms of price increments are distributed according to Gauss's law. And here the question arises: what is the mechanism of transition of the market from the state of normal functioning to the state of excitement and, perhaps, pre-crisis state.

The model we propose is based on the following postulates.

1. The process of information transfer is quantum. i.e. the information is transmitted in portions, multiples of a quantum of information. There are discrete information levels. When moving from one level to another, it is necessary to absorb or emit one quantum of information. Thus, the amount of information of a particular level is necessarily a multiple of the quantum of information. The situation is somewhat similar to that in physics, where there is a quantum of "action" (Planck's constant h) and the amount of emitted or absorbed energy is always proportional to h ($E = h\nu$, where ν – the frequency of radiation). In physics, h is a very small quantity. We cannot estimate the value of h_0 (so in the future we will denote the quantum of information), but the only thing that can be said for sure - it is quite a large value.
2. The probability density function for each information price level $p_n(y) = \Psi_n \cdot \Psi_n = \Psi_n^2(y)$, where $\Psi(y)$ is called the wave function of prices.
3. Wave function of prices can be defined as the solution of the equation:

$$\frac{d^2\Psi}{dy^2} + \frac{2}{h_0^2} (E - \frac{\omega^2 y^2}{2}) \Psi = 0 \tag{4}$$

with certain boundary conditions.

Here

$$E = \frac{\omega y^2}{2} + \frac{y^2}{2} = \frac{\omega^2 B^2}{2} \cos^2 \omega t + \frac{\omega^2 B^2}{2} \sin^2 \omega t = \frac{\omega^2 B^2}{2} = \text{const} > 0$$

where y is defined by the relation (3).

Equation (4) is an analogue of E. Schrödinger's quantum mechanical equation for determining the wave function of a quantum oscillator, where E is the total energy of the oscillator (kinetic plus potential) (Landau and Lifshitz, 1977). In our formulation of the problem E is the full amount of information determined by the level of return of the asset y , including both internal information of the system (market) and external information in relation to the market. Assuming in (4)

$$\alpha = \frac{2E}{h_0^2}; \beta = \frac{\omega}{h_0}; \frac{\alpha}{\beta} = \lambda = \frac{2E}{h_0\omega}$$

and introducing a new variable $\mu = y\sqrt{\beta}$, we give (4) the form

$$\Psi'' + (\lambda - \mu^2) \cdot \Psi = 0 \tag{5}$$

Here $\Psi'' = \frac{d^2\Psi}{d\mu^2}$.

First of all, we find the asymptotic behavior of the wave function of prices. When $\mu \rightarrow \pm\infty$ λ compared to μ^2 can be neglected $\Psi'' - \mu^2 \cdot \Psi = 0$. The solution of this equation satisfying the condition $\lim_{\mu \rightarrow \infty} \Psi = 0$ will

be $\Psi_\infty = e^{-\frac{\mu^2}{2}}$. The General solution (6) we find for in

the form $\Psi = \Psi_\infty \cdot u = e^{-\frac{\mu^2}{2}} \cdot u$. The condition of tending Ψ to zero while $y \rightarrow \pm\infty$ is satisfied only if $\lambda = 2n + 1$ ($n=0, 1, 2 \dots$). From where, taking into account the previously entered notation $\lambda = \frac{2E}{H_0\omega}$, we obtain

$$E_n = h_0\omega(n + \frac{1}{2}) \tag{6}$$

Only under this condition, the wave function of prices at infinity turns to zero. Then the solution of equation (5) and hence equation (4) has the following form

$$\Psi_n = C_n e^{-\frac{\mu^2}{2}} \cdot H_n(\mu) \tag{7}$$

Herewith μ is related to y by formula $\mu = y\sqrt{\beta} = \frac{y}{y_0}$,

$$y_0 = \frac{1}{\beta}$$

Here $H_n(\mu)$ the n -th degree Hermite polynomial. In the closed form, Hermite polynomials are defined by the relation $H_n(\mu) = (-1)^n \cdot e^{\mu^2} \cdot \frac{d^n(e^{-\mu^2})}{d\mu^n}$. In particular

$H_0(\mu) = 1$; $H_1(\mu) = 2\mu$; $H_2(\mu) = 4\mu^2 - 2$; $H_3(\mu) = 8\mu^3 - 12\mu$. The coefficient C_n is determined from the

normalization condition $\int_{-\infty}^{\infty} P_n(y) dy = \int_{-\infty}^{\infty} \Psi_n^2(y) dy = 1$.

Finally

$$\Psi_n(y) = \frac{1}{\sqrt{\sqrt{\pi} \cdot 2^n n! y_0}} \cdot e^{-\frac{1}{2}(\frac{y}{y_0})^2} \cdot H_n(\frac{y}{y_0}) \tag{8}$$

Behavior of the wave function of prices is illustrated by Figure 1.

The graph shows that in the potential well region ($E > V$) the solutions for Ψ have the type of harmonic functions. In the area outside the potential barrier ($E < V$), the solutions will contain two parts: exponentially decreasing and exponentially increasing. It is obvious that the solution of the problem is reduced to finding such conditions under which exponentially

increasing solution will be absent. This is possible only at discrete levels of information.

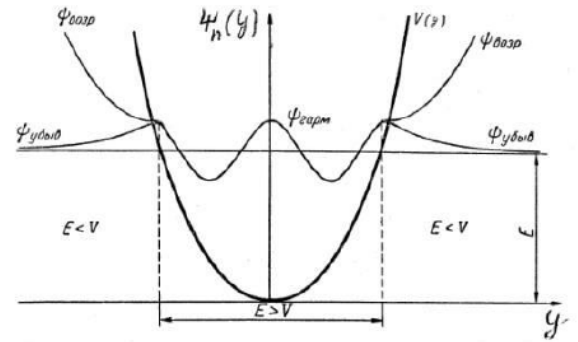


Figure 1: The wave function of a quantum harmonic price oscillator at an arbitrary value of information.

In the domain of small quantum numbers, the functions have the following form.

1. $n = 0$.

$$E_0 = h_0\omega/2; \Psi_0 = \frac{1}{\sqrt{\sqrt{\pi}y_0}} e^{-\frac{1}{2}(y/y_0)^2};$$

$$P_0(y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}(y/\sigma)^2}, \sigma = y_0/\sqrt{2}.$$

Thus, for $n=0$, the random function $y = y(t)$ for each value of t is distributed with a probability density function $P_0(y)$ (normal distribution).

2. $n = 1$.

$$E_1 = \frac{3h_0\omega}{2}; \Psi_1 = \frac{2}{\sqrt{2\sqrt{\pi}y_0}} (\frac{y}{y_0}) e^{-\frac{1}{2}(y/y_0)^2};$$

$$P_1(y) = \frac{2}{\sqrt{\pi}y_0} (\frac{y}{y_0})^2 e^{-(y/y_0)^2}$$

3. $n = 2$.

$$E_2 = \frac{5h_0\omega}{2}; \Psi_2 = \frac{(2(y/y_0)^2 - 1)}{\sqrt{2\sqrt{\pi}y_0}} e^{-\frac{1}{2}(y/y_0)^2};$$

$$P_2(y) = \frac{(2(y/y_0)^2 - 1)^2}{\sqrt{\pi}2y_0} e^{-(y/y_0)^2}.$$

From Figure 2 it is seen that the probability density distribution $P_n(y) = \Psi_n^2(y)$ will differ significantly from the probability density function of the classical harmonic price oscillator. Only in the field of large quantum numbers ($n \rightarrow \infty$), as expected, $\Psi_n^2 \rightarrow P(y)$, that is, the probability density distribution of the quantum harmonic oscillator (CGO) passes into the probability density distribution of the classical oscillator (Figure 3).

The frequency of radiation or absorption of information that determines the price of an asset in the

transition from one information level to another is determined by the ratio arising from the formula (6)

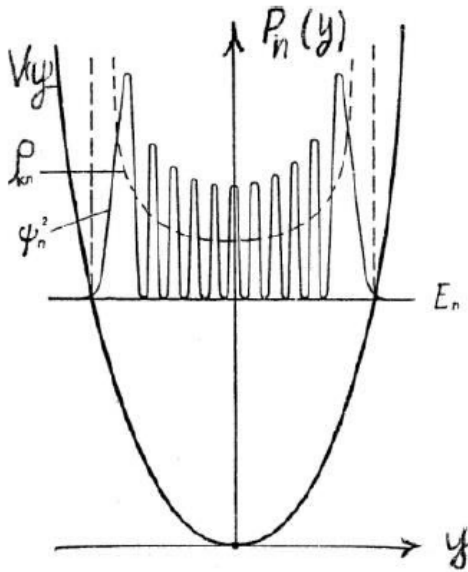


Figure 2: Graphs of probability density functions of quantum harmonic oscillator of prices at small values of quantum numbers.

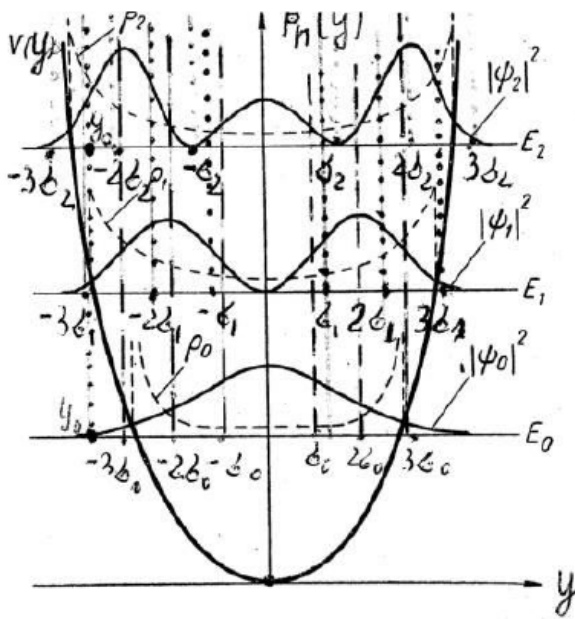


Figure 3: Comparison of the results of quantum and classical theories for the quantum harmonic oscillator of prices in the region of large values of quantum numbers (here $n=10$).

$$\omega_{mn} = (E_n - E_m) / h_0.$$

The expectation for the quantum harmonic oscillator of the financial market is defined as

$$M(y) = C_n^2 \int_{-\infty}^{\infty} y e^{-(y/y_0)^2} H_n^2(y/y_0) dy$$

It is easy to show that for any n , $M(y) = 0$. The quantum harmonic oscillator variance is determined by the relation:

$$D(y) = \sigma^2(y) = C_n^2 \int_{-\infty}^{\infty} y^2 e^{-(y/y_0)^2} H_n^2(y/y_0) dy$$

Here $C_n = (2^n n! \sqrt{\pi} y_0)^{-1/2}$ is normalization factor. As a result of calculations we find

$$\sigma^2(y_n) = D(y_n) = (2n+1) \frac{y_0^2}{2}$$

Hence standard deviation equals

$$\sigma(y_n) = \sqrt{2n+1} \cdot \frac{y_0}{\sqrt{2}} = \sqrt{2n+1} \cdot \sigma_{HOPM}. \text{ In particular.}$$

$$\text{For } n=0 \quad D(y_0) = 1/2 y_0^2; \quad \sigma(y_0) = \sigma_{HOPM} = 1/\sqrt{2} y_0$$

$$\text{For } n=1 \quad D(y_1) = 3/2 y_0^2; \quad \sigma(y_1) = \sqrt{3} \cdot \sigma_{HOPM}.$$

$$\text{For } n=2 \quad D(y_2) = 5/2 y_0^2; \quad \sigma(y_2) = \sqrt{5} \cdot \sigma_{HOPM}.$$

Let us illustrate what has been said. The probability of the logarithm of price increment is in a given interval for the quantum harmonic oscillator is determined by the ratio

$$P_n \{y_1 < y < y_2\} = \frac{1}{\sqrt{\pi} 2^n n! y_0} \int_{y_1}^{y_2} e^{-(y/y_0)^2} H_n^2(y/y_0) dy \quad (9)$$

Assuming $y_1 = -\sigma_n$; $y_2 = \sigma_n$, we find the probability that the value of random value y deviates from the expectation in one direction or another by the value of the standard deviation. If $n=0$ (normal distribution). Here $H_0=1$ $n=0$, $P\{-\sigma_0 < y < \sigma_0\} = \Phi(y_2/\sigma_0) - \Phi(y_1/\sigma_0) = 2\Phi(1) = 0,6826$, where $\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-t^2/2} dt$

– Laplace function; $\sigma_0 = \sigma_{HOPM}$. Assuming $y_1 = -2\sigma_0$; $y_2 = 2\sigma_0$, we find $P\{-2\sigma_0 < y < 2\sigma_0\} = 2\Phi(2) = 0,9544$.

If $y_1 = -3\sigma_0$; $y_2 = 3\sigma_0$ we have $P\{-3\sigma_0 < y < 3\sigma_0\} = 3\Phi(3) = 0,9973$, that is, the rule of "three Sigma": the probability that the value of a normally distributed random value belongs to the interval $(-3\sigma_0; 3\sigma_0)$ is almost equal to one. If $n=1$; $H_1(y/y_0) = 2(y/y_0)$;

By (9) we have

$$P\{y_1 < y < y_2\} = \left\{ \Phi\left(\frac{y_2}{\sigma_0}\right) - \Phi\left(\frac{y_1}{\sigma_0}\right) \right\} - \frac{1}{\sqrt{\pi}} \left\{ \frac{y_2}{\sqrt{2}\sigma_0} e^{-\left(\frac{y_2}{\sqrt{2}\sigma_0}\right)^2} - \frac{y_1}{\sqrt{2}\sigma_0} e^{-\left(\frac{y_1}{\sqrt{2}\sigma_0}\right)^2} \right\}$$

If $y_1 = -\sigma_0$; $y_2 = \sigma_0$;

$$P(-\sigma_0 < y < \sigma_0) = \Phi(1) - \Phi(-1) - \frac{1}{\sqrt{\pi}} \frac{1}{\sqrt{2}} e^{-\frac{1}{2}} \approx 0,1986.$$

$$y_1 = -2\sigma_0; y_2 = 2\sigma_0;$$

$$P(-2\sigma_0 < y < 2\sigma_0) = \Phi(2) - \Phi(-2) - \frac{1}{\sqrt{\pi}} \frac{3\sqrt{2}}{e^2} \approx 0,7384.$$

$$y_1 = -3\sigma_0; y_2 = 3\sigma_0; P(-3\sigma_0 < y < 3\sigma_0) = \Phi(3) - \frac{1}{\sqrt{\pi}} \frac{2\sqrt{2}}{e^{4,5}} \approx 0,9706.$$

i.e. the probability that the value of random value does not fall into the interval $(-3\sigma_0; 3\sigma_0)$ is quite noticeable value ≈ 0.03 . In other words, at $n=1$ the rule of "three" normal "Sigma" already "does not work" and the event, extremely rare from the point of view of Gauss distribution, becomes rather expected at the information excited price levels ($n=1,2,3,\dots$).

However, the probability estimations of deviations from the mathematical expectation becomes close to the usual one if we take the standard deviation for the level as a measure of the spread we'll take the standard deviation for the level $n=1$:

$$\sigma_1 = \sqrt{3}\sigma_{HOPM} = \sqrt{3}\sigma_0.$$

If $y_1 = -\sigma_1; y_2 = \sigma_1$ we find

$$P\{-\sigma_1 < y < \sigma_1\} = 2\Phi(\sqrt{3}) - \frac{\sqrt{6}}{\sqrt{\pi}e^{1,5}} \approx 0,608 \quad \text{if } y_1 = -2\sigma_1, y_2 = 2\sigma_1 \text{ we have } P\{-2\sigma_1 < y < 2\sigma_1\} = 0,9960.$$

If $y_1 = -3\sigma_1, y_2 = 3\sigma_1$ we have

$$P\{-3\sigma_1 < y < 3\sigma_1\} = 0,9999.$$

That is, the rule of "three sigma" works, but not for the Gauss distribution, but for the distribution of the level $n=1$ (Figure 2).

Let's consider $n=2$. Here $H_2(y/y_0) = 4(y/y_0) - 2$. By formula (9) we have $P(y_1 - y_2) = \left\{ \Phi\left(\frac{y_2}{\sigma_0}\right) - \Phi\left(\frac{y_1}{\sigma_0}\right) \right\} - \frac{1}{\sqrt{\pi}} \left\{ \left(\frac{y_2}{\sqrt{2}\sigma_0}\right)^2 e^{-\left(\frac{y_2}{\sqrt{2}\sigma_0}\right)^2} - \left(\frac{y_1}{\sqrt{2}\sigma_0}\right)^2 e^{-\left(\frac{y_1}{\sqrt{2}\sigma_0}\right)^2} \right\} - \frac{1}{2\sqrt{\pi}} \left\{ \left(\frac{y_2}{\sqrt{2}\sigma_0}\right)^2 e^{-\left(\frac{y_2}{\sqrt{2}\sigma_0}\right)^2} - \left(\frac{y_1}{\sqrt{2}\sigma_0}\right)^2 e^{-\left(\frac{y_1}{\sqrt{2}\sigma_0}\right)^2} \right\}$

If $y_1 = -\sigma_0; y_2 = \sigma_0;$

$$P(-\sigma_0 < y < \sigma_0) = 2\Phi(1) - \frac{2}{\sqrt{\pi}} \left(\frac{1}{\sqrt{2}}\right)^3 e^{-\left(\frac{1}{\sqrt{2}}\right)^2} - \frac{1}{\sqrt{\pi}} \frac{2}{\sqrt{2}} e^{-\left(\frac{1}{\sqrt{2}}\right)^2} \approx 0,1987.$$

If $y_1 = -2\sigma_0; y_2 = 2\sigma_0$

$$P(-2\sigma_0 < y < 2\sigma_0) = 2\Phi(2) - \frac{2}{\sqrt{\pi}} (\sqrt{2})^3 e^{-2} - \frac{1}{\sqrt{\pi}} \sqrt{2} e^{-2} \approx 0,4145.$$

If $y_1 = -3\sigma_0; y_2 = 3\sigma_0$

$$P(-3\sigma_0 < y < 3\sigma_0) = 2\Phi(3) - \frac{2}{\sqrt{\pi}} \left(\frac{3}{\sqrt{2}}\right)^3 e^{-4,5} - \frac{1}{\sqrt{\pi}} \frac{3}{\sqrt{2}} e^{-4,5} \approx 0,8682.$$

So, the probability that the value of random value does not belong to the interval $(-3\sigma_0; 3\sigma_0)$ equals to ≈ 0.1328 . At $n=2$ the rule of "three" normal "Sigma" "does not work" even more than at $n=1$ and the event, the rarest for the normal distribution, is quite expected here.

However, the probabilistic estimate of the deviations from the expectation at $n=2$ becomes relatively close to the estimate at $n=0$ (Gauss distribution) if we take the standard deviation $\sigma_2 = \sqrt{5} \cdot \sigma_0$ for the level $n=2$ as a measure of the spread.

$$P\{-\sigma_2 < y < \sigma_2\} = 0,3705; P\{-2\sigma_2 < y < 2\sigma_2\} = 0,9785; P\{-3\sigma_2 < y < 3\sigma_2\} = 0,99999998.$$

In other words, the "three Sigma" rule also works for the $n=2$ level distribution.

This is illustrated in Figure 2. The deviation of the price logarithm increment from the expected value to the value y_0 at $n=2$ (E_2 axis) is not a very rare event, realized with a probability of ≈ 0.02 . However, from the point of view of the normal distribution $n=0$ (E_0 axis), this deviation from the mathematical expectation is a rare event, realized with a probability $\approx 10^{-7}$.

In the Bachelier-Samuelson financial world, in which the increments of the logarithms of prices are distributed according to Gauss law, all events are scaled by the fundamental unit of measurement, the standard deviation σ_0 . In this regard, it becomes clear what is "normal" and what is "abnormal" according to the Gaussian model. The fall in prices on the USA stock market 19.10.1987 by 22.6% and the rebound 21.10.1987 by 9.7% according to Gauss-events that should not happen. They should be impossible.

The fact that they have occurred says that the market can deviate significantly from the norm. These events are "outliers", they lie "outside" of what is possible for the rest of the set of increments. The probability of falling prices by 22.6% is about 10^{-7} , which brings this phenomenon far beyond the range of $3\sigma_{HOPM}$. According to estimates of [6], the waiting time for such an event is ~ 7 thousand years. We have the same order of magnitude time estimate for the repetition of each of the three largest "emissions" on the American stock market in the 20th century (1914, 1929, 1987), a total of ~ 3 trillion years. In fact, three crashes occurred in the same century. This suggests that financial emissions form their own class, which is manifested in their statistical characteristics, they differ from the rest of the population, forming a dynamic price series, and for their explanation require a new, different from the Gaussian model. This is relevant, if only because the market crash that occurs simultaneously

in most stock markets around the world, as demonstrated in October 1987 or autumn 2008, represents an almost instantaneous "evaporation" of trillions of dollars.

IV. CONCLUSIONS

The approach developed in this paper, which explains the possibility of before mentioned conflicts, seems to be more harmonious than the traditional approaches considered in the first part of the article. We consider a model based on the assumption of the quantum (discontinuous) nature of the impact of information on financial markets. As a consequence, the value of assets, changing, consistently moves from one discrete level to another, and each of them corresponds to a certain amount of information provision $E_n (n=0,1,2,...)$. The hierarchy of information-price levels is autonomous in the sense that each of them has its own separate probabilistic constitution – different functions of probability density distribution. The normal distribution takes place only when $n=0$. For all others $n=1, 2, 3...$ the density functions are different from Gaussian. As a consequence, the probability of event that the logarithm of the price increment belongs to a given interval with the growth of n is increasingly different from that calculated under the assumption of fairness for a given level of normal distribution. The scale of risk measurement on the "price line", is determined by the standard deviation $\sigma(y_n) = \sqrt{2n+1} \cdot \sigma_{HOPM}$ ($\sigma \approx \sqrt{2n}$ in the field of large quantum numbers). This phenomenon, in our opinion, should be called "quantum volatility". Thus, the event, the rarest in the normal distribution scale, has a sufficiently noticeable probability already at the scales of excited levels E2 or E3. This means that in information-rich, volatile financial markets, price emissions are realistically expected.

The impact of central bank key rate and commercial banks credit rates on creating and maintaining of a favorable investment climate (Brusov, Filatova, Orekhova, Kulik, Weil and Brailov 2018) from point of view of quantum nature of information. From this point of view can be studied also the process of inflation and formation of a consumer basket (Popov 2018).

The probability density function $P_n(y) = \Psi_n^2$ (where, $P_n(y) = \Psi_n^2$ defined by formula (8), is the solution of equation (4)) has a number of attractive properties.

First: this distribution has all moments $v_k = M(Y^k) = \int_{-\infty}^{\infty} y^k P_n(y) dy$ ($k=1, 2, 3, 4...$) in particular the variance of the random variable Y is finite. (The property is highly desirable, allowing to avoid huge difficulties in statistical analysis).

Second: the distribution $P_n(y)$ at a fixed n is stable, with a stability index $\alpha=2$ (the same as that of the normal distribution) determining the behavior of

random value $Y = \{\dots y_k \dots\}$ at infinity (asymptotics). However, the value of the tail index $\alpha=2$, the same as that of the normal distribution, would seem to indicate the absence of "heavy tails". However, this is not so, because with the growth of n (the growth of information saturation of the market), the scale of risk measurement on the price scale changes (increases) $\sigma(y_n) \approx \sqrt{2n}$ and the absence of heavy tails for the information excited level $n=2$ on the scale of $n=0$ means their appearance on the asymptotics (Figure 2).

Third: the finiteness and difference from zero of the fourth moment $v_4 = M(Y^4)$ predetermines the presence of a significant excess, growing with increasing n , which also indicates the appearance of heavy tails on the asymptotic scale of the normal distribution.

Fourth: the equality of the third moment to zero $v_3 = M(Y^3) = 0$ indicates the absence of asymmetry (skewness) in the distribution $P_n(y) = \Psi_n^2$, which, it seems, is not confirmed by the graphs of empirical distribution densities (histograms). But the output of the value of the increment of the logarithm of prices beyond the boundaries of the interval $\pm 2\sigma$, and even more $\pm 3\sigma$ so-this is a rare event in the market. That is, the sample of extreme values, which is the only way to evaluate the "tail effects" such as the bevel coefficient or stability index may well be not representative, not representing the features of the population. If, however, the asymmetry of the real distribution does occur, it means that equation (4) must be "corrected" taking into account the presence of asymmetry in the solution, but necessarily preserving the idea of the quantum nature of the information.

REMARK

In our opinion there is the possibility for further development of the approach, suggested in this paper. It would be interesting to study more precisely the quantum of information value. Perhaps it is possible to obtain an algorithm of its estimation.

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